Ember

EDUCATION

MF27

H2 MATHEMATICS

Fees & Referral

Fees

Year 1	$340 / month
Year 2	$350 / month
Year 3	$360 / month

First 50 sign-ups get 20% off their first month.

Taking 2 or more subjects? $50 off your second subject.

Referral Programme

Know a friend who hates Maths too? Refer them.

1st referral	$88 cash
2nd referral	$108 cash
3rd referral	$138 cash
4th referral	$188 cash
5th referral	$288 cash

Your referred friend also gets 20% off their first month (first 50 sign-ups only).

Algebraic Series

Binomial expansion

$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \cdots + b^n,$ where n is a positive integer

and $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$

Maclaurin expansion

$$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}\,f''(0) + \cdots + \frac{x^n}{n!}\,f^{(n)}(0) + \cdots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}\,x^2 + \cdots + \frac{n(n-1)\cdots(n-r+1)}{r!}\,x^r + \cdots \quad (|x| < 1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^r}{r!} + \cdots \quad (\text{all } x)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^r\,x^{2r+1}}{(2r+1)!} + \cdots \quad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^r\,x^{2r}}{(2r)!} + \cdots \quad (\text{all } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + \frac{(-1)^{r+1}\,x^r}{r} + \cdots \quad (-1 < x \leq 1)$$

Partial Fractions Decomposition

Non-repeated linear factors:

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

Repeated linear factors:

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{ax+b} + \frac{B}{cx+d} + \frac{C}{(cx+d)^2}$$

Non-repeated quadratic factor:

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{ax+b} + \frac{Bx+C}{x^2+c^2}$$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A \equiv 2 \sin A \cos A$$
$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2\cos^2 A - 1 \equiv 1 - 2\sin^2 A$$
$$\tan 2A \equiv \frac{2\tan A}{1 - \tan^2 A}$$

Principal values

$$-\tfrac{1}{2}\pi \leq \sin^{-1} x \leq \tfrac{1}{2}\pi \quad (|x| \leq 1) \qquad 0 \leq \cos^{-1} x \leq \pi \quad (|x| \leq 1) \qquad -\tfrac{1}{2}\pi < \tan^{-1} x < \tfrac{1}{2}\pi$$

Derivatives

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$

Integrals

Arbitrary constants are omitted; a denotes a positive constant.

$f(x)$	$\int f(x)\,\mathrm{d}x$	
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$	$\lvert x\rvert < a$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$	$x > a$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert$	$\lvert x\rvert < a$
$\tan x$	$\ln(\sec x)$	$\lvert x\rvert < \frac{1}{2}\pi$
$\cot x$	$\ln(\sin x)$	$0 < x < \pi$
$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x + \cot x)$	$0 < x < \pi$
$\sec x$	$\ln(\sec x + \tan x)$	$\lvert x\rvert < \frac{1}{2}\pi$

Vectors

The point dividing AB in the ratio $\lambda : \mu$ has position vector $\frac{\mu\mathbf{a}+\lambda\mathbf{b}}{\lambda+\mu}$.

Vector product

$$\mathbf{a}\times\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

Applications of Definite Integrals

Arc length of a curve defined in Cartesian form:

$$\int_a^b \sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$$

Surface area of revolution about the x-axis for a curve defined in Cartesian form:

$$\int_a^b 2\pi y\sqrt{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x$$

Functions of Two Variables

Quadratic approximation of f at (a, b):

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b) + \tfrac{1}{2} f_{xx}(a,b)(x-a)^2 + f_{xy}(a,b)(x-a)(y-b) + \tfrac{1}{2} f_{yy}(a,b)(y-b)^2$$

Numerical Methods

Trapezium rule (for single strip):

$$\int_a^b f(x)\,\mathrm{d}x \approx \tfrac{1}{2}(b-a)\left[f(a) + f(b)\right]$$

Simpson's rule (for two strips):

$$\int_a^b f(x)\,\mathrm{d}x \approx \tfrac{1}{6}(b-a)\left[f(a) + 4f\left(\tfrac{a+b}{2}\right) + f(b)\right]$$

The Newton-Raphson iteration for approximating a root of $f(x) = 0$:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

where x_1 is a first approximation.

Euler method with step size h

$$y_2 = y_1 + h\,f(x_1, y_1)$$

Improved Euler method with step size h

$$u_2 = y_1 + h\,f(x_1, y_1)$$
$$y_2 = y_1 + \tfrac{h}{2}\left[f(x_1, y_1) + f(x_2, u_2)\right]$$

Standard Discrete Distributions

Distribution of X	$P(X = x)$	Mean	Variance
Binomial B(n, p)	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson Po(λ)	$e^{-\lambda}\frac{\lambda^x}{x!}$	λ	λ
Geometric Geo(p)	$(1-p)^{x-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$

Standard Continuous Distribution

Distribution of X	p.d.f.	Mean	Variance
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

Sampling and Testing

Unbiased estimate of population variance

$$s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{(\sum x)^2}{n}\right] = \frac{n}{n-1} \cdot \frac{\sum(x-\bar{x})^2}{n}$$

Regression and Correlation

Estimated product moment correlation coefficient

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \cdot \sum(y-\bar{y})^2}} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)\left(\sum y^2 - \frac{(\sum y)^2}{n}\right)}}$$

Estimated regression line of y on x

$y - \bar{y} = b(x - \bar{x})$, where $b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

Mathematical Results

AM-GM inequality

For any nonnegative real numbers $x_1, x_2, \ldots, x_n$,

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \cdots x_n}$$

where equality holds if and only if $x_1 = x_2 = \cdots = x_n$.

Cauchy-Schwarz inequality

For any real numbers $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$,

$$\left(\sum_{i=1}^{n} u_i v_i\right)^2 \leq \left(\sum_{i=1}^{n} u_i^2\right)\left(\sum_{i=1}^{n} v_i^2\right)$$

where equality holds if and only if there exists a nonzero constant k such that $u_i = kv_i$ for all $i = 1, 2, \ldots, n$.

Triangle inequality

For any real numbers $x_1, x_2, \ldots, x_n$,

$$|x_1 + x_2 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|$$

where equality holds if $x_1, x_2, \ldots, x_n$ are all nonnegative.

Inclusion-Exclusion Principle

For any subsets $A_1, A_2, \ldots, A_n$ of a set,

$$\begin{aligned}
|A_1 \cup A_2 \cup \cdots \cup A_n| = {} & |A_1| + |A_2| + \cdots + |A_n| \\
& - \big[|A_1 \cap A_2| + |A_1 \cap A_3| + \cdots + |A_{n-1} \cap A_n|\big] \\
& + \big[|A_1 \cap A_2 \cap A_3| + \cdots + |A_{n-2} \cap A_{n-1} \cap A_n|\big] \\
& \vdots \\
& + (-1)^{n-1}|A_1 \cap A_2 \cap \cdots \cap A_n|
\end{aligned}$$

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