



COMMONLY ASKED
QUESTIONS !!

Equations & Inequalities

H2 MATHEMATICS

**◆ EQUATIONS & INEQUALITIES****◆ Commonly Asked Question 1.**

Without the use of a calculator, solve the inequality

$$\frac{3x^2 - 4x - 7}{x^2 - x - 6} \geq 2.$$

Hence solve the inequality

$$\frac{3x^2 - 4|x| - 7}{x^2 - |x| - 6} \geq 2.$$

Solution.

★ Approach!

- DO NOT cross multiply. In H2 math, form one fraction.

Put the inequality into one fraction before factorising.

$$\begin{aligned} \frac{3x^2 - 4x - 7}{x^2 - x - 6} - 2 &\geq 0. \\ \frac{3x^2 - 4x - 7 - 2(x^2 - x - 6)}{x^2 - x - 6} &\geq 0. \\ \frac{3x^2 - 4x - 7 - 2x^2 + 2x + 12}{x^2 - x - 6} &\geq 0. \end{aligned}$$

Complete the square in the numerator because it cannot be factorised.

$$\begin{aligned} \frac{x^2 - 2x + 5}{x^2 - x - 6} &\geq 0. \\ \frac{(x - 1)^2 - 1^2 + 5}{(x - 3)(x + 2)} &\geq 0. \\ \frac{(x - 1)^2 + 4^{(1)}}{(x - 3)(x + 2)} &\geq 0. \end{aligned}$$

(I) If numerator or denominator cannot be factorised. Simply complete the square.

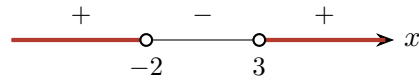
Since $(x - 1)^2 \geq 0$, $(x - 1)^2 + 4 \geq 4$, so $(x - 1)^2 + 4$ is always positive.



Since the numerator is positive,
for the whole fraction to be positive, the denominator is also positive.

However, the denominator is always exclusive of 0.

The critical values are -2 and 3 , both hollow.



$$\therefore x < -2 \text{ or } x > 3.$$

Remember

- Always write this statement and solve till the equation to be solved is shown!
- Always use a number line to show working!
- Final ans will never be AND. Always OR.

$$\frac{3x^2 - 4x - 7}{x^2 - x - 6} \geq 2$$

Replace x with $|x|$:

$$\frac{3|x|^2 - 4|x| - 7}{|x|^2 - |x| - 6} \geq 2.$$

$$\frac{3x^2 - 4|x| - 7}{x^2 - |x| - 6} \geq 2.$$

This uses $|x|^2 = x^2$.

From above, $x < -2$ or $x > 3$.

$$\therefore |x| < -2 \text{ (rej as } |x| \geq 0) \text{ or } |x| > 3.$$

$$\therefore x > 3 \text{ or } x < -3.$$

Remember

- In a hence part, write the substituted inequality out in full first. Here that is the substitution line with $|x|$ written in place of every x .
- Jumping from the first answer straight to $|x| > 3$ loses the method mark, even when the final answer is right!

**◆ Commonly Asked Question 2.**

(i) Without using a calculator, solve the inequality

$$\frac{5x-1}{x^2+x-6} \leq 1.$$

(ii) Hence solve

$$1 + \frac{1-5\ln x}{(\ln x)^2 + \ln x - 6} \geq 0.$$

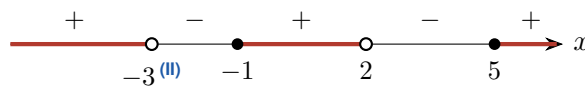
Solution.

(i)

$$\begin{aligned} \frac{5x-1}{x^2+x-6} &\leq 1. \\ \frac{5x-1}{x^2+x-6} - 1 &\leq 0. \\ \frac{5x-1-(x^2+x-6)}{x^2+x-6} &\leq 0. \\ \frac{5x-1-x^2-x+6}{x^2+x-6} &\leq 0. \\ \frac{-x^2+4x+5}{x^2+x-6} &\leq 0. \\ \frac{-[x^2-4x-5]}{x^2+x-6} &\stackrel{(i)}{\leq} 0. \end{aligned}$$

(I) Remove the negative sign. Remember to flip inequality sign!

$$\begin{aligned} \frac{x^2-4x-5}{x^2+x-6} &\geq 0. \\ \frac{(x-5)(x+1)}{(x+3)(x-2)} &\geq 0. \end{aligned}$$



(II) Hollow for exclusive full for inclusive.

$$\therefore x < -3 \text{ or } -1 \leq x < 2 \text{ or } x \geq 5.$$



(ii)

$$\frac{5x - 1}{x^2 + x - 6} \leq 1$$

Replace x with $\ln x$:

$$\frac{5(\ln x) - 1}{(\ln x)^2 + (\ln x) - 6} \leq 1.$$

$$\frac{-[1 - 5 \ln x]}{(\ln x)^2 + \ln x - 6} \leq 1.$$

$$\frac{1 - 5 \ln x}{(\ln x)^2 + \ln x - 6} \geq -1.$$

$$1 + \frac{1 - 5 \ln x}{(\ln x)^2 + \ln x - 6} \geq 0.$$

From (i), $x < -3$ or $-1 \leq x < 2$ or $x \geq 5$.

$$\therefore \ln x < -3 \text{ or } -1 \leq \ln x < 2 \text{ or } \ln x \geq 5.$$

$$0^{(\text{III})} < x < e^{-3} \text{ or } e^{-1} \leq x < e^2 \text{ or } x \geq e^5.$$

(III) x cannot be ≤ 0 for $\ln x$ to exist!

**◆ Commonly Asked Question 3.**

(a) Using an algebraic method, solve the inequality

$$\frac{x+4}{3+2x-x^2} < 1.$$

(b) Hence solve the inequality

$$\frac{x^2-4}{x^4+2x^2-3} < 1.$$

Solution.

(a)

$$\frac{x+4}{3+2x-x^2} < 1.$$

$$\frac{x+4}{3+2x-x^2} - 1 < 0.$$

$$\frac{x+4-(3+2x-x^2)}{3+2x-x^2} < 0.$$

$$\frac{x+4-3-2x+x^2}{3+2x-x^2} < 0.$$

$$\frac{x^2-x+1}{3+2x-x^2} < 0.$$

$$\frac{x^2-x+1}{-(x^2-2x-3)} < 0.$$

$$\frac{x^2-x+1}{x^2-2x-3} > 0.$$

$$\frac{(x-\frac{1}{2})^2 - (\frac{1}{2})^2 + 1}{(x-3)(x+1)} > 0.$$

$$\frac{(x-\frac{1}{2})^2 + \frac{3}{4}}{(x-3)(x+1)} > 0.$$

Since $(x-\frac{1}{2})^2 \geq 0$, $(x-\frac{1}{2})^2 + \frac{3}{4} \geq \frac{3}{4}$.

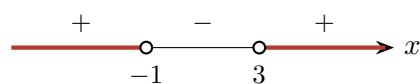
$$\left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0.$$

Since the numerator is positive,

for the whole fraction to be positive, the denominator is also positive.

However, the denominator is always exclusive of 0.

The critical values are -1 and 3 , both hollow.



$$x < -1 \text{ or } x > 3.$$



(b)

$$\frac{x+4}{3+2x-x^2} < 1$$

Replace x with $-x^2$:

$$\frac{-x^2+4}{3+2(-x^2)-(-x^2)^2} < 1.$$

$$\frac{-x^2+4}{3-2x^2-x^4} < 1.$$

$$\frac{x^2-4}{-(3-2x^2-x^4)} < 1.$$

$$\frac{x^2-4}{x^4+2x^2-3} < 1.$$

From (a), $x < -1$ or $x > 3$.

$$\therefore -x^2 < -1 \text{ or } -x^2 > 3.$$

$$x^2 > 1 \text{ or } x^2 < -3 \text{ (rej as } x^2 \geq 0).$$

$$x < -1 \text{ or } x > 1.$$

**◆ Commonly Asked Question 4.**

Given that a is a positive real number, solve the inequality

$$\frac{x^2 - x + 2}{x^2 + (1 - a)x - a} \leq 0,$$

leaving your answer in terms of a .

Solution.

$$\frac{x^2 - x + 2}{x^2 + (1 - a)x - a} \leq 0.$$

Factorise the denominator, including the unknown a .

$$\frac{(x - \frac{1}{2})^2 - (\frac{1}{2})^2 + 2}{(x - a)(x + 1)} \leq 0.$$

(I) Try to see the factorisation including an unknown.

$$\frac{(x - \frac{1}{2})^2 + \frac{7}{4}}{(x - a)(x + 1)} \leq 0.$$

Since $(x - \frac{1}{2})^2 \geq 0$, $(x - \frac{1}{2})^2 + \frac{7}{4} \geq \frac{7}{4}$.

$$\left(x - \frac{1}{2}\right)^2 + \frac{7}{4} > 0.$$

Since the numerator is positive,

for the whole fraction to be negative, the denominator is negative.

However, the denominator is always exclusive of 0.

The critical values are -1 and a , both hollow. Since $a > 0$, a is to the right of -1 .



$$-1 < x < a.$$

**◆ Commonly Asked Question 5.**

Without the use of a calculator, solve the inequality

$$\frac{2x^2 + 2x - 11}{x^2 - 2x + 1} \geq 1.$$

Hence solve the inequality

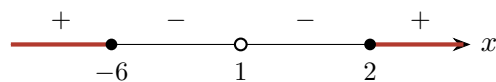
$$\frac{11x^2 - 2x - 2}{x^2 - 2x + 1} \leq -1.$$

Solution.

$$\begin{aligned}\frac{2x^2 + 2x - 11}{x^2 - 2x + 1} &\geq 1. \\ \frac{2x^2 + 2x - 11}{x^2 - 2x + 1} - 1 &\geq 0. \\ \frac{2x^2 + 2x - 11 - (x^2 - 2x + 1)}{x^2 - 2x + 1} &\geq 0. \\ \frac{2x^2 + 2x - 11 - x^2 + 2x - 1}{x^2 - 2x + 1} &\geq 0. \\ \frac{x^2 + 4x - 12}{x^2 - 2x + 1} &\geq 0. \\ \frac{(x + 6)(x - 2)}{(x - 1)^2} &\geq 0.\end{aligned}$$

The critical values are -6 , 1 and 2 .

$(x - 1)^2 > 0$ for $x \neq 1$, so the sign does not change at 1 .



$$\therefore x \leq -6 \text{ or } x \geq 2.$$

Remember

- Trickiest type. Be careful!



$$\frac{2x^2 + 2x - 11}{x^2 - 2x + 1} \geq 1$$

Replace x with $\frac{1}{x}$:

$$\frac{2\left(\frac{1}{x}\right)^2 + 2\left(\frac{1}{x}\right) - 11}{\left(\frac{1}{x}\right)^2 - 2\left(\frac{1}{x}\right) + 1} \geq 1.$$

$$\frac{\frac{2}{x^2} + \frac{2}{x} - 11}{\frac{1}{x^2} - \frac{2}{x} + 1} \geq 1.$$

$$\frac{2 + 2x - 11x^2}{1 - 2x + x^2} \geq 1.$$

$$-\frac{11x^2 - 2x - 2}{x^2 - 2x + 1} \geq 1.$$

$$\frac{11x^2 - 2x - 2}{x^2 - 2x + 1} \leq -1.$$

From above, $x \leq -6$ or $x \geq 2$.

$$\therefore \frac{1}{x} \leq -6 \text{ or } \frac{1}{x} \geq 2.$$

$$\frac{1}{x} = -6 \text{ at } x = -\frac{1}{6}, \text{ and } \frac{1}{x} = 2 \text{ at } x = \frac{1}{2}.$$

By GC, plot $y = \frac{1}{x}$ with the lines $y = -6$ and $y = 2$.

Steps:

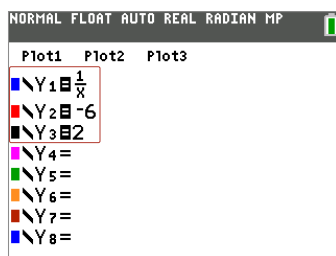
1) Press $Y=$, enter:

$$Y_1 = \frac{1}{X}$$

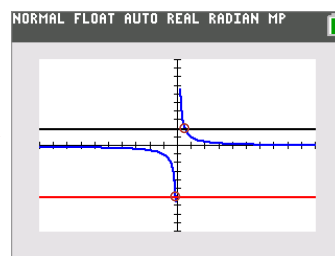
$$Y_2 = -6$$

$$Y_3 = 2$$

2) Plot the graph.



GRAPH



From the GC, the curve is on or below $y = -6$, or on or above $y = 2$, for:

$$\therefore -\frac{1}{6} \leq x < 0 \text{ or } 0 < x \leq \frac{1}{2} \text{ (I)}.$$

(I) For $\frac{1}{x}$, check the upper limit if they are the same.

When $x = 0$,

$$\frac{11(0)^2 - 2(0) - 2}{0^2 - 2(0) + 1} = -2 \leq -1.$$

$$\therefore -\frac{1}{6} \leq x \leq \frac{1}{2}.$$